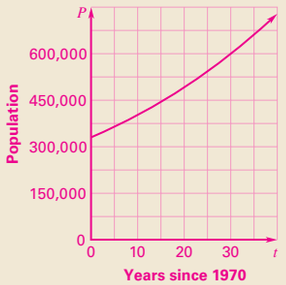
Extra Example 4

In 1970, the population of Kern County, California, where Bakersfield is located, was about 330,000. From 1970 to 2000, the county population grew at an average annual rate of about 2.4%.

- Write an exponential growth model giving the population P of Kern County t years after 1970.

  About how many people lived in Kern County in 1990?
  $P = 330{,}000(1.024)^t$; about 530,000 people
- Graph the model.

Kern County California



- Use the graph to estimate the year when the population of Kern County was about 400,000. **1980**

Key Questions to Ask for Example 4

- Will the number of computer security incidents continue to grow at 92% a year? Explain.
  Possible answer: No; the rate of growth will probably slow down considerably as protection against viruses is developed.

Animated Algebra classzone.com

An **Animated Algebra** activity is available on-line for **Example 4**. This activity is also available on the **Power Presentations CD-ROM**.

Vocabulary

If students are having trouble remembering the difference between the growth factor and the percent increase, have them write "growth factor = 1 + percent increase" in their notebooks for reference.

EXPONENTIAL GROWTH MODELS When a real-life quantity increases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$y = a(1 + r)^t$$

where a is the initial amount and r is the percent increase expressed as a decimal. Note that the quantity $1 + r$ is the growth factor.

EXAMPLE 4 Solve a multi-step problem

COMPUTERS In 1996, there were 2573 computer viruses and other computer security incidents. During the next 7 years, the number of incidents increased by about 92% each year.

- Write an exponential growth model giving the number n of incidents t years after 1996. About how many incidents were there in 2003?
- Graph the model.
- Use the graph to estimate the year when there were about 125,000 computer security incidents.

Solution

STEP 1 The initial amount is $a = 2573$ and the percent increase is $r = 0.92$. So, the exponential growth model is:

$n = a(1 + r)^t$ **Write exponential growth model.**

$= 2573(1 + 0.92)^t$ **Substitute 2573 for a and 0.92 for r.**

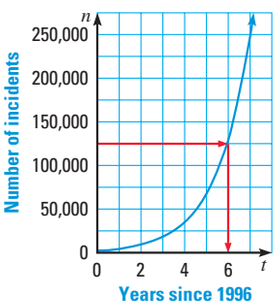
$= 2573(1.92)^t$ **Simplify.**

AVOID ERRORS Notice that the percent increase and the growth factor are two different values. An increase of 92% corresponds to a growth factor of 1.92.

Using this model, you can estimate the number of incidents in 2003 ($t = 7$) to be $n = 2573(1.92)^7 \approx 247{,}485$.

STEP 2 The graph passes through the points (0, 2573) and (1, 4940.16). Plot a few other points. Then draw a smooth curve through the points.

STEP 3 Using the graph, you can estimate that the number of incidents was about 125,000 during 2002 ($t \approx 6$).



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GUIDED PRACTICE for Example 4

4. **WHAT IF?** In Example 4, estimate the year in which there were about 250,000 computer security incidents. **2003**
5. In the exponential growth model $y = 527(1.39)^x$, identify the initial amount, the growth factor, and the percent increase. **527, 1.39, 39%**

COMPOUND INTEREST Exponential growth functions are used in real-life situations involving *compound interest*. Compound interest is interest paid on the initial investment, called the *principal*, and on previously earned interest. Interest paid only on the principal is called *simple interest*.

KEY CONCEPT *For Your Notebook*

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

EXAMPLE 5 Find the balance in an account

FINANCE You deposit $4000 in an account that pays 2.92% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. Quarterly

b. Daily

Solution

a. With interest compounded quarterly, the balance after 1 year is:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Write compound interest formula.

$= 4000\left(1 + \frac{0.0292}{4}\right)^{4 \cdot 1}$ $P = 4000$, $r = 0.0292$, $n = 4$, $t = 1$

$= 4000(1.0073)^4$ Simplify.

≈ 4118.09 Use a calculator.

▸ The balance at the end of 1 year is $4118.09.

b. With interest compounded daily, the balance after 1 year is:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Write compound interest formula.

$= 4000\left(1 + \frac{0.0292}{365}\right)^{365 \cdot 1}$ $P = 4000$, $r = 0.0292$, $n = 365$, $t = 1$

$= 4000(1.00008)^{365}$ Simplify.

≈ 4118.52 Use a calculator.

▸ The balance at the end of 1 year is $4118.52.

GUIDED PRACTICE for Example 5

6. FINANCE You deposit $2000 in an account that pays 4% annual interest. Find the balance after 3 years if the interest is compounded daily. **$2254.98**

Differentiated Instruction

Kinesthetic Learners The concept of compound interest will be new to some students. Before presenting them with the formula for calculating the amount in an interest-earning account over time, choose simple values for P, r, and n. Have groups of students explore how to calculate the interest earned for various values of t. Then, after they have learned the formula, have them verify their results.

See also the *Algebra 2 Toolkit* for more strategies.

Extra Example 5

You deposit $5500 in an account that pays 3.6% annual interest. Find the balance after 2 years if interest is compounded with the given frequency.

a. semiannually **$5908.70**

b. monthly **$5909.97**

Key Questions to Ask for Example 5

- Without doing any calculations, what do you know about what the balance would be in Example 5 if interest were compounded monthly? **It would be greater than $4118.09 and less than $4118.52.**
- In compound interest, why does the amount of interest earned increase as the frequency of compounding increases? **The more often interest is compounded, the sooner the new interest is added to the balance on which interest is earned.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What does the graph of an exponential growth function look like?

- **An exponential growth function is a function of the form $y = ab^x$ with $a > 0$ and $b > 1$.**
- **Exponential growth models, such as the compound interest formula, describe situations in which a quantity increases by a fixed percent each time period.**

The graph of an exponential growth function of the form $y = ab^x$ is a curve that rises from left to right and gets steeper as x increases. The x-axis is a horizontal asymptote of the graph.